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CLARK-OCONE TYPE FORMULAS IN THE MEIXNER WHITE NOISE ANALYSIS

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In the classical Gaussian analysis the Clark-Ocone formula allows to reconstruct an integrand if we know the Itô stochastic integral. This formula can be written in the form

$$F = \mathbf{E}F + \int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\} dW_t,$$

where a function (a random variable) F is square integrable with respect to the Gaussian measure and differentiable by Hida; $\mathbf{E}$ — the expectation; $\mathbf{E}\{\circ|_{\mathcal{F}_t}\}$ — the conditional expectation with respect to a full σ-algebra $\mathcal{F}_t$ that is generated by the Wiener process W up to the point of time t; $\partial_{\cdot} F$ — the Hida derivative of F; $\int \circ(t) dW_t$ — the Itô stochastic integral with respect to the Wiener process.

In this paper we explain how to reconstruct an integrand in the case when instead of the Gaussian measure one considers the so-called generalized Meixner measure μ (depending on parameters, μ can be the Gaussian, Poissonian, Gamma measure etc.) and obtain corresponding Clark-Ocone type formulas.

Introduction

Denote by $\mathcal{D}$ the Schwartz space of infinite-differentiable real-valued functions on $\mathbb{R}_+ := [0, +\infty)$ with compact supports; by $\mathcal{D}'$ the distribution space that is dual of $\mathcal{D}$; by $\langle\cdot,\cdot\rangle$ the pairing between elements of $\mathcal{D}'$ and $\mathcal{D}$, this pairing is generated by the scalar product in the space of square integrable with respect to the Lebesgue measure functions on $\mathbb{R}_+$; by the subindex $\mathbb{C}$ complexifications of spaces. The notation $\langle\cdot,\cdot\rangle$ will be preserved for pairing in tensor powers and complexifications of spaces.

Let μ be the standard Gaussian measure on $(\mathcal{D}', C(\mathcal{D}'))$ (here and below $C(\mathcal{D}')$ is the σ-algebra on $\mathcal{D}'$ that is generated by cylindrical sets), i.e., a probability measure with the Laplace transform

$$l_\mu(\lambda) = \int_{\mathcal{D}'} e^{\langle x,\lambda\rangle} \mu(dx) = e^{\langle\lambda,\lambda\rangle/2}, \quad \lambda \in \mathcal{D}_{\mathbb{C}}.$$

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As it is well known (e.g., [6, 25, 21]), any square integrable with respect to μ and differentiable by Hida complex-valued function F on $\mathcal{D}'$ can be presented in the form

$$F = \mathbf{E}F + \int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}dW_t, \tag{1}$$

where $\mathbf{E}$ is the expectation and $\mathbf{E}\{\circ|_{\mathcal{F}_t}\}$ is the conditional expectation with respect to a full σ-algebra $\mathcal{F}_t$ that is generated by the Wiener process W up to the point of time t, i.e., $\mathcal{F}_t = \sigma(W_s : s \le t)$; $\partial_\cdot F$ — the Hida derivative of F and $\int \circ(t)dW_t$ — the Itô stochastic integral with respect to W (usually for stochastic integrals on $\mathbb{R}_+$ we do not write limits of integration for simplification of notations). Formula (1) is called the *Clark-Ocone formula*. As we can see, this formula allows us to reconstruct a version of the integrand (this integrand is not unique, generally speaking) if we know the result of stochastic integration.

As it is known (e.g., [8, 30]), formula (1) holds true (up to clear modifications) if instead of the Gaussian measure one considers the Poissonian one. Moreover, one can easily avoid a restrictive assumption that F must be differentiable by Hida: it is sufficient to generalize the Clark-Ocone formula to spaces of generalized functions (see, e.g., [7, 9]).

Clark-Ocone formulas and their generalizations (in this paper they will be called *Clark-Ocone type formulas*) have applications in the stochastic analysis and in the financial mathematics, see, e.g., [18, 4, 9, 23, 10, 26, 22, 12, 8, 30] and references therein. In order to satisfy demands of applications (for example, in some problems it is necessary to reconstruct an integrand by the result of integration, in another problems it is necessary to recontsruct a random variable by the family of conditional expectations of its stochastic derivative, etc.), different variants of such formulas on various spaces, with different stochastic derivatives and with stochastic integrals with respect to various random processes and measures were obtained, see, in particular, [19, 21, 4, 7, 5, 20, 9, 22, 30, 8]. For example, in [21, 20] a Clark-Ocone type formula that is connected with Lévy processes was obtained, this formula contains stochastic integrals with respect to a Wiener process and with respect to a compensated Poissonian random measure. In [9] an another way of construction of Clark-Ocone type formulas that are connected with Lévy processes was offered, this way is based on the Nualart-Schoutens representation for a square integrable random variable [24, 28]; now the Clark-Ocone type formulas contain integrals with respect to special random processes. Moreover, these formulas were obtained in [9] not only for square integrable random variables, but also for generalized ones.

In this paper we obtain Clark-Ocone type formulas in the so-called Meixner white noise analysis. This analysis is connected with the generalized Meixner measure μ [27] (see also Subsection 1.1) which, depending on parameters, can be the Gaussian, Poissonian, Gamma measure etc., and with the corresponding Meixner random process M (the derivative of which is the Meixner white noise that is connected with μ). Note that under some assumptions (see Subsection 1.3) M is a Lévy process. Nevertheless, our constructions essentially differ from the constructions of [21, 20] and [9]: we try to preserve a "classical" form of Clark-Ocone type formulas and therefore exploit a Hida stochastic derivative and stochastic integrals with respect to M only. Of course, in the particular cases when μ is the Gaussian or Poissonian measure, our formulas reduce to the corresponding classical Clark-Ocone formulas.

The paper is organized in the following manner. In the first section we recall necessary definitions and results (the generalized Meixner measure, properties of the corresponding space of square integrable functions, the extended (Skorohod) stochastic integral, the Hida stochastic derivative, properties of these operators). In the second section we deal with Clark-Ocone type formulas and related matters. Note that here we obtain these formulas on the space of square integrable with respect to the generalized Meixner measure functions only, the case of spaces of generalized functions will be considered in another paper.

1 Preliminaries

1.1 The generalized Meixner measure

Let us define the generalized Meixner measure (see [27] for more details and explanations). Let $\rho, \nu : \mathbb{R}_+ \to \mathbb{C}$ be smooth functions such that

$$\theta \stackrel{def}{=} \rho - \nu : \mathbb{R}_+ \to \mathbb{R}, \quad \eta \stackrel{def}{=} \rho\nu : \mathbb{R}_+ \to \mathbb{R}_+ \tag{2}$$

and, moreover, θ and η are bounded on $\mathbb{R}_+$. Further, for each $t \in \mathbb{R}_+$ let $v_{\rho(t),\nu(t)}(ds)$ be a probability measure on $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$ (here $\mathcal{B}(\mathbb{R})$ is the Borel σ-algebra on $\mathbb{R}$) which is defined by its Fourier transform

$$\int_{\mathbb{R}} e^{i\zeta s} v_{\rho(t),\nu(t)}(ds) = \exp\Big\{ - i\zeta\big(\rho(t) + \nu(t)\big) +$$
$$2\sum_{m=1}^{\infty} \frac{\big(\rho(t)\nu(t)\big)^m}{m} \Big[\sum_{n=2}^{\infty} \frac{(-i\zeta)^n}{n!}\big(\nu^{n-2}(t) + \nu^{n-3}(t)\rho(t) + \cdots + \rho^{n-2}(t)\big)\Big]^m\Big\}.$$

Definition. *A probability measure μ on the measurable space $(\mathcal{D}', C(\mathcal{D}'))$ with the Fourier transform*

$$\int_{\mathcal{D}'} e^{i\langle x,\xi\rangle}\mu(dx) = \exp\Big\{\int_{\mathbb{R}_+} dt \int_{\mathbb{R}} v_{\rho(t),\nu(t)}(ds)\frac{1}{s^2}\big(e^{is\xi(t)} - 1 - is\xi(t)\big)\Big\}$$

is called the generalized Meixner measure.

Depending on parameters ρ and ν, μ we can get, in particular, the Gaussian, Poissonian, Pascal, Meixner or Gamma measure.

It was proved in [27] that the generalized Meixner measure μ is the measure of a generalized random process [11] with independent values; and the Laplace transform $l_\mu(\cdot) = \int_{\mathcal{D}'} \exp\{\langle x, \cdot\rangle\}\mu(dx)$ of μ is a holomorphic at $0 \in \mathcal{D}_{\mathbb{C}}$ function.

1.2 The space of square integrable functions

Let $(L^2) := L^2(\mathcal{D}', \mu)$ be the space of complex-valued square integrable with respect to the generalized Meixner measure μ functions on $\mathcal{D}'$. We construct now a natural orthogonal basis in (L^2). For $n \in \mathbb{N}$ denote by $\overline{\mathcal{P}}_n$ the closure in (L^2) of the set of all continuous polynomials on $\mathcal{D}'$ of degree $\le n$, $\overline{\mathcal{P}}_0 := \mathbb{C}$. Denote also $(L^2_n) := \overline{\mathcal{P}}_n \ominus \overline{\mathcal{P}}_{n-1}$ (the orthogonal complemention in (L^2)), $(L_0^2) := \mathbb{C}$. Since μ has a holomorphic at zero Laplace transform, the set of continuous polynomials on $\mathcal{D}'$ is dense in (L^2) [29], therefore $(L^2) = \bigoplus_{n=0}^{\infty}(L_n^2)$.

Denote by $\widehat{\otimes}$ a symmetric tensor product. For each $f^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}n}$, $n \in \mathbb{Z}_+$ ($\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}0} := \mathbb{C}$), we define $:\langle x^{\otimes n}, f^{(n)}\rangle:$ as the orthogonal projection of $\langle x^{\otimes n}, f^{(n)}\rangle$ onto (L_n^2), $x \in \mathcal{D}'$. It follows from results of [27] that $:\langle x^{\otimes n}, f^{(n)}\rangle: = \langle P_n(x), f^{(n)}\rangle$, where $P_n(x) \in \mathcal{D}'^{\widehat{\otimes}n}$ are the kernels of (generalized Appell) polynomials with a generating function $\gamma(\lambda)\exp\{\langle x, \alpha(\lambda)\rangle\}$, $\lambda \in \mathcal{D}_{\mathbb{C}}$, i.e.,

$$\gamma(\lambda)\exp\{\langle x, \alpha(\lambda)\rangle\} = \sum_{n=0}^{\infty}\frac{1}{n!}\langle P_n(x), \lambda^{\otimes n}\rangle,$$

where $\alpha(\lambda) = \lambda + \sum_{n=2}^{\infty}\frac{\lambda^n}{n}(\rho^{n-1} + \rho^{n-2}\nu + \cdots + \nu^{n-1})$ and

$$\gamma(\lambda) = \frac{1}{l_\mu(\alpha(\lambda))} = \exp\left\{-\int_{\mathbb{R}_+}\Big(\frac{\lambda^2(t)}{2} + \sum_{n=3}^{\infty}\frac{\lambda^n(t)}{n}\big(\rho^{n-2}(t) + \rho^{n-3}(t)\nu(t) + \cdots + \nu^{n-2}(t)\big)\Big)dt\right\}.$$

Let us define (*real*, i.e., *bilinear*) scalar products $\langle\cdot,\cdot\rangle_{ext}$ on $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}n}$, $n \in \mathbb{Z}_+$, by setting for $f^{(n)}, g^{(n)} \in \mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}n}$

$$\langle f^{(n)}, g^{(n)}\rangle_{ext} := \frac{1}{n!}\int_{\mathcal{D}'}\langle P_n(x), f^{(n)}\rangle\langle P_n(x), g^{(n)}\rangle\mu(dx).$$

It follows from results of [27] that

$$\begin{aligned}
\langle f^{(n)}, g^{(n)}\rangle_{ext} = &\sum_{\substack{k,l_j,s_j\in\mathbb{N}:\ j=1,\dots,k,\ l_1>l_2>\dots>l_k,\\ l_1s_1+\dots+l_ks_k=n}}\frac{n!}{l_1^{s_1}\dots l_k^{s_k}s_1!\dots s_k!}\times\\
&\int_{\mathbb{R}_+^{s_1+\dots+s_k}} f^{(n)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1},\dots,t_{s_1}}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k}}_{l_k})\times\\
&g^{(n)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1},\dots,t_{s_1}}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k}}_{l_k})\eta^{l_1-1}(t_1)\dots\eta^{l_1-1}(t_{s_1})\times\\
&\eta^{l_2-1}(t_{s_1+1})\dots\eta^{l_2-1}(t_{s_1+s_2})\dots\eta^{l_k-1}(t_{s_1+\dots+s_{k-1}+1})\dots\eta^{l_k-1}(t_{s_1+\dots+s_k})\,dt_1\dots dt_{s_1+\dots+s_k}.
\end{aligned} \tag{3}$$

So, for example, for $n = 1$ $\langle f^{(1)}, g^{(1)}\rangle_{ext} = \langle f^{(1)}, g^{(1)}\rangle = \int_{\mathbb{R}_+} f^{(1)}(t)g^{(1)}(t)dt$, for $n = 2$ $\langle f^{(2)}, g^{(2)}\rangle_{ext} = \langle f^{(2)}, g^{(2)}\rangle + \int_{\mathbb{R}_+} f^{(2)}(t,t)g^{(2)}(t,t)\eta(t)dt$. If (see (2)) $\eta = 0$ (the case of Gaussian or Poissonian μ) then $\langle f^{(n)}, g^{(n)}\rangle_{ext} = \langle f^{(n)}, g^{(n)}\rangle$, in the general case $\langle f^{(n)}, g^{(n)}\rangle_{ext} = \langle f^{(n)}, g^{(n)}\rangle + \dots$.

Let $|\cdot|_{ext}$ denotes the norm which is generated by the scalar product $\langle\cdot,\cdot\rangle_{ext}$, i.e., for $n \in \mathbb{Z}_+$ $|f^{(n)}|_{ext} := \sqrt{\langle f^{(n)}, \overline{f^{(n)}}\rangle_{ext}}$. Denote by $\mathcal{H}_{ext}^{(n)}$ the Hilbert space which is the closure of $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes}n}$ with respect to $|\cdot|_{ext}$ (in particular, $\mathcal{H}_{ext}^{(0)} = \mathbb{C}$).

Let $\mathcal{H} := L^2(\mathbb{R}_+)$ be the space of complex-valued square integrable with respect to the Lebesgue measure functions on $\mathbb{R}_+$. It is clear that $\mathcal{H}_{ext}^{(1)} = \mathcal{H}$. For $n \in \mathbb{N}\backslash\{1\}$ the space $\mathcal{H}_{ext}^{(n)}$ can be understood as an extension of $\mathcal{H}^{\widehat{\otimes} n}$ in a generalized sense: let $F^{(n)} \in \mathcal{H}^{\widehat{\otimes} n}$, $f^{(n)} \in F^{(n)}$ be a representative (a function) from the equivalence class $F^{(n)}$ with a "zero diagonal", i.e., $f^{(n)}(t_1, \ldots, t_n) = 0$ if there exist $i, j \in \{1, \ldots, n\}$ such that $i \neq j$ but $t_i = t_j$. It is easy to show ([16]), that the function $f^{(n)}$ generates an equivalence class in $\mathcal{H}_{ext}^{(n)}$ which can be identified with $F^{(n)}$.

Note that, of course, the space $\mathcal{H}_{ext}^{(n)}$ depends on the parametric function η, see (2) (for example, if $\eta = 0$ then $\mathcal{H}_{ext}^{(n)} = \mathcal{H}^{\widehat{\otimes} n}$), but we do not use η in the designation of this space for simplification of notation.

For $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{Z}_+$, we define a polynomial $\langle P_n, F^{(n)} \rangle \in (L^2)$ as

$$\langle P_n, F^{(n)} \rangle := (L^2)\text{-}\lim_{k \to \infty} \langle P_n, f_k^{(n)} \rangle,$$

where $\mathcal{D}_{\mathbb{C}}^{\widehat{\otimes} n} \ni f_k^{(n)} \to F^{(n)}$ in $\mathcal{H}_{ext}^{(n)}$ as $k \to \infty$ (this difinition is well-posed, as is easy to verify). The forthcoming statement easily follows from the construction of polynomials $\langle P_n, F^{(n)} \rangle$ (see also [27]).

Theorem. *A function $F \in (L^2)$ if and only if there exists a sequence of kernels*

$$\big(F^{(n)} \in \mathcal{H}_{ext}^{(n)}\big)_{n=0}^{\infty}$$

such that F can be presented in the form

$$F = \sum_{n=0}^{\infty} \langle P_n, F^{(n)} \rangle, \tag{4}$$

where the series converges in (L^2), i.e., the (L^2)-norm of F

$$\|F\|_{(L^2)}^2 = \sum_{n=0}^{\infty} n! |F^{(n)}|_{ext}^2 < \infty. \tag{5}$$

Moreover, the system $\big\{\langle P_n, F^{(n)} \rangle,\ F^{(n)} \in \mathcal{H}_{ext}^{(n)},\ n \in \mathbb{Z}_+\big\}$ is an orthogonal basis in (L^2) in the sense that for $F, G \in (L^2)$ of form (4) the (real) scalar product in (L^2)

$$(F, G)_{(L^2)} = \sum_{n=0}^{\infty} n! \langle F^{(n)}, G^{(n)} \rangle_{ext}.$$

1.3 The extended stochastic integral

By analogy with the Gaussian analysis, on the probability triplet $(\mathcal{D}', C(\mathcal{D}'), \mu)$ we define the Meixner random process M by setting for each $t \in \mathbb{R}_+$ $M_t := \langle P_1, 1_{[0,t)} \rangle \in (L^2)$, here and below $1_B(y)$ is the indicator of the event $\{y \in B\}$.

Remark. *If the parametric functions ρ and ν (see Subsection 1.1) are constants then M is a Lévy process; but, in general, it is not the case (M can be a not time-homogeneous process).*

Using results of [27] one can show that M *is a locally square integrable normal martingale (with respect to the generated by M flow of full σ-algebras) with orthogonal independent increments*, therefore one can consider the Itô stochastic integral with respect to M.

Let us recall the construction of the extended (Skorohod) stochastic integral with respect to M (see [16] for details). Let $G \in (L^2) \otimes \mathcal{H}$. It follows from above-posed results that G can be presented in the form

$$G(\cdot) = \sum_{n=0}^{\infty} \langle P_n, G_{\cdot}^{(n)} \rangle, \tag{6}$$

$G_{\cdot}^{(n)} \in \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$, with $\|G\|^2_{(L^2)\otimes\mathcal{H}} = \sum\limits_{n=0}^{\infty} n! |G_{\cdot}^{(n)}|^2_{\mathcal{H}_{ext}^{(n)}\otimes\mathcal{H}} < \infty$.

If in addition G is such that the kernels $G_{\cdot}^{(n)}$ belong to $\mathcal{H}^{\widehat{\otimes} n} \otimes \mathcal{H} \subset \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$ (the embedding in the generalized sense described above) then one can show [16] that F can be presented in the form

$$G(\cdot) = \sum_{n=0}^{\infty} n! \int_0^{\infty} \int_0^{t_n} \dots \int_0^{t_2} G_{\cdot}^{(n)}(t_1, \dots, t_n) dM_{t_1} \dots dM_{t_n}, \tag{7}$$

i.e., as a series of repeated Itô stochastic integrals with respect to the Meixner process. In this case one can define the extended stochastic integral of G with respect to M as

$$\begin{gathered}\int G(t)\widehat{d}M_t := \sum_{n=0}^{\infty} (n+1)! \int_0^{\infty} \int_0^{t} \int_0^{t_n} \dots \int_0^{t_2} \widehat{G}^{(n)}(t_1, \dots, t_n, t) dM_{t_1} \dots dM_{t_n} dM_t = \\ \sum_{n=0}^{\infty} \langle P_{n+1}, \widehat{G}^{(n)} \rangle \in (L^2)\end{gathered} \tag{8}$$

(cf. [2, 3, 1]), where $\widehat{G}^{(n)} \in \mathcal{H}^{\widehat{\otimes} n+1} \subset \mathcal{H}_{ext}^{(n+1)}$ are the projections of $G_{\cdot}^{(n)}$ onto $\mathcal{H}^{\widehat{\otimes} n+1}$, if this series converges in (L^2). Note that if in addition G is integrable by Itô then series (8) is the result of term by term integration of series (7), the convergence of (8) in (L^2) follows in this case from the condition $G \in (L^2) \otimes \mathcal{H}$.

For a general $G \in (L^2) \otimes \mathcal{H}$ the above mentioned definition cannot be accepted because it is impossible to project elements of $\mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$ onto $\mathcal{H}_{ext}^{(n+1)}$, generally speaking. Nevertheless, the following natural generalization is possible. Let $G_{\cdot}^{(n)} \in \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$. We select a representative (a function) $g_{\cdot}^{(n)} \in G_{\cdot}^{(n)}$ with the property $g_t^{(n)}(t_1, \dots, t_n) = 0$ if there exists $j \in \{1, \dots, n\}$ such that $t_j = t$. Let us define the element $\widehat{G}^{(n)} \in \mathcal{H}_{ext}^{(n+1)}$ as the equivalence class in $\mathcal{H}_{ext}^{(n+1)}$ that is generated by the symmetrization of $g_{\cdot}^{(n)}$ with respect to $n+1$ variables (note that for $n = 0$ we have $\mathcal{H}_{ext}^{(0)} \otimes \mathcal{H} = \mathcal{H} \ni G_{\cdot}^{(0)} = \widehat{G}^{(0)} \in \mathcal{H} = \mathcal{H}_{ext}^{(1)}$). It was proved in [16] that $\widehat{G}^{(n)}$ is well-defined and $|\widehat{G}^{(n)}|_{ext} \leq |G_{\cdot}^{(n)}|_{\mathcal{H}_{ext}^{(n)}\otimes\mathcal{H}}$.

Definition. *Let $G \in (L^2) \otimes \mathcal{H}$ and be such that $\sum\limits_{n=0}^{\infty} (n+1)! |\widehat{G}^{(n)}|^2_{ext} < \infty$, where the elements $\widehat{G}^{(n)} \in \mathcal{H}_{ext}^{(n+1)}$ were constructed above by the kernels $G_{\cdot}^{(n)} \in \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$ from decomposition (6) for G. We define the extended stochastic integral with respect to M $\int G(t)\widehat{d}M_t \in (L^2)$ by setting*

$$\int G(t)\widehat{d}M_t := \sum_{n=0}^{\infty} \langle P_{n+1}, \widehat{G}^{(n)} \rangle.$$

In particular cases, when the generalized Meixner measure μ is the Gaussian or Poissonian one, the operator $\int \circ(t)\widehat{d}M_t$ is the classical extended Skorohod stochastic integral [2, 3, 1]. The forthcoming statement explains why we preserve this term in a general case.

Theorem. *([16]) Let $G \in (L^2) \otimes \mathcal{H}$ and be integrable by Itô with respect to M (i.e., be adapted with respect to the generated by M flow of σ-algebras). Then G is integrable in the extended sense, and $\int G(t)\widehat{d}M_t = \int G(t)dM_t$ (the last integral is the Itô one).*

1.4 The Hida stochastic derivative

Finally, let us recall the notion of the Hida stochastic derivative in the Meixner white noise analysis (see [14, 15] for more details). First we note that, as it was proved in [16], any $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}$, can be considered as an element $F^{(n)}(\cdot)$ of the space $\mathcal{H}_{ext}^{(n-1)} \otimes \mathcal{H}$, and $|F^{(n)}(\cdot)|_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}} \leq |F^{(n)}|_{ext}$.

Definition. *Let $F \in (L^2)$ and be such that*

$$\sum_{n=1}^{\infty} n!n|F^{(n)}(\cdot)|^2_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}} < \infty, \tag{9}$$

where $F^{(n)}(\cdot)$ are the kernels from decomposition (4) for F, in point as elements of $\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}$. We define the Hida stochastic derivative $\partial_\cdot F \in (L^2) \otimes \mathcal{H}$ by setting

$$\partial_\cdot F := \sum_{n=1}^{\infty} n\langle P_{n-1}, F^{(n)}(\cdot)\rangle.$$

Theorem. *([16]) The extended stochastic integral $\int \circ(t)\widehat{d}M_t : (L^2) \otimes \mathcal{H} \to (L^2)$ and the Hida stochastic derivative $\partial_\cdot : (L^2) \to (L^2)\otimes\mathcal{H}$ are adjoint one to another, and, in particular, are closed operators.*

2 Clark-Ocone type formulas and related matters

2.1 A Clark-Ocone formula in the simplest particular case and problems of the general case

Let μ be the generalized Meixner measure on $(\mathcal{D}', C(\mathcal{D}'))$. In the case when μ is not the Gaussian or Poissonian measure ($\eta \neq 0$, see (2)), but $F \in (L^2)$ is differentiable by Hida and is such that all kernels $F^{(n)}$ from decomposition (4) belong to $\mathcal{H}^{\widehat{\otimes}n}$ (now we consider $\mathcal{H}^{\widehat{\otimes}n}$ as a subspace of $\mathcal{H}_{ext}^{(n)}$ in the generalized sense described in Subsection 1.2), the analog of classical Clark-Ocone formula (1) has a form

$$F = \mathbf{E}F + \int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}dM_t, \tag{10}$$

where the notation as in (1) up to obvious modifications (for example, $\mathcal{F}_t = \sigma(M_s : s \leq t)$). Using the definitions of the Hida stochastic derivative, of the extended stochastic integral and the fact that for an integrable by Itô integrand this integral coincides with the Itô one, of the expectation ($\mathbf{E}F = \int_{\mathcal{D}'} F(x)\mu(dx)$), and the fact that for $F \in (L^2)$ of form (4)

$$\mathbf{E}\{F|_{\mathcal{F}_t}\} = \langle P_0, F^{(0)}\rangle + \sum_{n=1}^{\infty}\langle P_n, F^{(n)}1_{[0,t)^n}\rangle \tag{11}$$

[17], one can conclude that (10) is valid if for any $n \in \mathbb{N}\backslash\{1\}$ and for each $F^{(n)} \in \mathcal{H}^{\widehat{\otimes} n}$

$$nPr\big(F^{(n)}(\cdot_1,\dots,\cdot_{n-1},\cdot_n)1_{[0,\cdot_n)^{n-1}}(\cdot_1,\dots,\cdot_{n-1})\big) = F^{(n)}$$

in $\mathcal{H}^{\widehat{\otimes} n}$, here and below Pr denotes a symmetrization with respect to all variables. But this equality is fulfilled in $\mathcal{H}^{\widehat{\otimes} n} = L^2(\mathbb{R}_+, m)^{\widehat{\otimes} n}$ (m is the Lebesgue measure on $\mathbb{R}_+$) because if $t_1,\dots,t_n$ are mutually *different* then $Pr1_{[0,t_n)^{n-1}}(t_1,\dots,t_{n-1}) = \frac{1}{n}$, and $m^{\otimes n}\big(\{t_1,\dots,t_n\} : \exists i,j \in \{1,\dots,n\} : i \neq j, t_i = t_j\big) = 0$. Note that one can prove (1) and its Poissonian counterpart by the same way.

In the general case not each $F \in (L^2)$ can be presented even in the form

$$F = \mathbf{E}F + \int G(t)\widehat{d}M_t, \tag{12}$$

$G \in (L^2) \otimes \mathcal{H}$ (see Proposition 2.1 below for details). But even if $F \in (L^2)$ is representable in form (12), formula (10) can be not valid. For example, let $F = \langle P_3, F^{(3)}\rangle$, $F^{(3)} \in \mathcal{H}^{(3)}_{ext}$. Then $\mathbf{E}F = 0$ and it is not difficult to calculate that

$$\int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}dM_t = \langle P_3, F^{(3)}(\cdot_1,\cdot_2,\cdot_3)\big(1_{[0,\cdot_3)^2}(\cdot_1,\cdot_2) + 1_{[0,\cdot_2)^2}(\cdot_3,\cdot_1) + 1_{[0,\cdot_1)^2}(\cdot_2,\cdot_3)\big)\rangle,$$

therefore using (5) and (3) we obtain

$$\begin{aligned}\|F - \int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}dM_t\|^2_{(L^2)} = 6|F^{(3)}\big(1 - [1_{[0,\cdot_3)^2}(\cdot_1,\cdot_2) + 1_{[0,\cdot_2)^2}(\cdot_3,\cdot_1) + 1_{[0,\cdot_1)^2}(\cdot_2,\cdot_3)]\big)|^2_{ext} = \\ 18\int_{\mathbb{R}^2_+} |F^{(3)}(t_1,t_1,t_2)|^2 1_{\{t_1\geq t_2\}}\eta(t_1)dt_1dt_2 + 12\int_{\mathbb{R}_+} |F^{(3)}(t_1,t_1,t_1)|^2\eta^2(t_1)dt_1.\end{aligned}$$

If $F^{(3)}$ is such that $\int_{\mathbb{R}_+} |F^{(3)}(t_1,t_1,t_1)|^2\eta^2(t_1)dt_1 = 0$ then $\langle P_3, F^{(3)}\rangle$ can be presented in form (12) (see Poposition 2.1 below), but, as we can see from the calculation above, even under this condition it is possible that $F \neq \int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}dM_t$.

Remark 2.1. *It is easy to understand intuitively why Clark-Ocone formula* (10) *is not valid in the general case: in order to calculate the norms in* $\mathcal{H}^{(n)}_{ext}$, $n > 2$, *one has to use nonsymmetrical functions, e.g.,* $Q(t_1,t_2) := F^{(3)}(t_1,t_1,t_2)$; *but applying the conditional expectation we "cut off" such functions and therefore lose an information.*

In what follows, we clarify a condition of representability of $F \in (L^2)$ in form (12), and explain how to reconstruct an integrand G.

2.2 A belonging of square integrable functions to the range of values of the extended stochastic integral

We begin from a simple example. Let $F = \langle P_2, F^{(2)}\rangle$, $F^{(2)} \in \mathcal{H}_{ext}^{(2)}$. It is clear that if F is representable in form (12) then $G(\cdot) = \langle P_1, G_\cdot^{(1)}\rangle$, $G_\cdot^{(1)} \in \mathcal{H}_{ext}^{(1)} \otimes \mathcal{H}$, and $F^{(2)} = \widehat{G}^{(1)}$ (see Subsection 1.3). But since by construction $\widehat{G}^{(1)}$ contains a representative $\widehat{g}^{(1)}$ such that for each $t \in \mathbb{R}_+$ $\widehat{g}^{(1)}(t,t) = 0$, we have a *necessary* condition of representability of $\langle P_2, F^{(2)}\rangle$ in form (12): $F^{(2)}$ must contain a representative $f^{(2)}$ such that for each $t \in \mathbb{R}_+$ $f^{(2)}(t,t) = 0$. Moreover, it is easy to see that this condition is *sufficient*: one can set $G_\cdot^{(1)} := F^{(2)}(\cdot)$ (i.e., we consider $F^{(2)}$ as an element of $\mathcal{H}_{ext}^{(1)} \otimes \mathcal{H}$).

In a general case the situation is quite similar. Namely, we have the following statement.

Proposition 2.1. *Let $F \in (L^2)$. The following statements are equivalent:*

(1) F can be presented in form (12) *with an integrand $G \in (L^2) \otimes \mathcal{H}$;*

(2) for each $n \in \mathbb{N}\backslash\{1\}$ the kernel $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$ from decomposition (4) *for F has a representative $f^{(n)}$ such that $f^{(n)}(t_1,\dots,t_n) = 0$ if for each $i \in \{1,\dots,n\}$ there exists $j \in \{1,\dots,n\}$ such that $i \neq j$, but $t_i = t_j$.*

Remark 2.2. *If, for example, $\eta = 0$ (see* (2)*) then for each $F \in (L^2)$ the condition of statement (2) is automatically fulfilled. In fact, it follows from* (3) *that considering properties of representatives of $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}\backslash\{1\}$, one can ignore families of arguments $\{t_1,\dots,t_n\}$ for which there exist $i,j \in \{1,\dots,n\}$ such that $i \neq j$, $t_i = t_j$, $\eta(t_i) = 0$ (i.e., one can redefine these representatives on described families of arguments in compliance with necessity).*

Proof. First we prove this proposition for $F = \langle P_n, F^{(n)}\rangle$, $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}\backslash\{1\}$.

1) ("(2)⇒(1)") Let $f^{(n)}$ be a representative of $F^{(n)}$ that is described in the condition of statement (2). Without loss of generality one can assume that $f^{(n)}$ is a symmetric function. We set

$$\begin{gathered} h_n(t_1,\dots,t_n) = Pr1_{\{t_1\neq t_n, t_2\neq t_n,\dots,t_{n-1}\neq t_n\}} = \\ \frac{1}{n}\left[1_{\{t_1\neq t_n,t_2\neq t_n,\dots,t_{n-1}\neq t_n\}} + 1_{\{t_n\neq t_{n-1},t_1\neq t_{n-1},\dots,t_{n-2}\neq t_{n-1}\}} + \dots + 1_{\{t_2\neq t_1,t_3\neq t_1,\dots,t_n\neq t_1\}}\right] \end{gathered} \tag{13}$$

(1_B is the indicator of the event B),

$$g_t^{(n-1)}(t_1,\dots,t_{n-1}) := \begin{cases} \frac{f^{(n)}(t_1,\dots,t_{n-1},t)}{h_n(t_1,\dots,t_{n-1},t)}, & \text{if } h_n(t_1,\dots,t_{n-1},t) \neq 0 \\ 0, & \text{if } h_n(t_1,\dots,t_{n-1},t) = 0 \end{cases} \tag{14}$$

(note that if $h_n(t_1,\dots,t_{n-1},t) = 0$ then $f^{(n)}(t_1,\dots,t_{n-1},t) = 0$ by the condition of statement (2)). Using (3), nonatomicity of the Lebesgue measure, the equality

$$h_n(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k}}_{l_k},t) = \frac{1}{n}1_{\{l_k>1\}} + \frac{s_k+1}{n}1_{\{l_k=1\}}$$

for *different* $t_1,\dots,t_{s_1+\dots+s_k},t$ (here $k,l_\cdot,s_\cdot\in\mathbb{N}$, $l_1>\dots>l_k$, $l_1s_1+\dots+l_ks_k=n-1$), and the condition from statement (2) (in the last inequality of the forthcoming calculation), we obtain

$$
\begin{aligned}
|g_\cdot^{(n-1)}|^2_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}} = &\sum_{\substack{k,l_j,s_j\in\mathbb{N}:\ j=1,\dots,k,\ l_1>l_2>\dots>l_k,\\ l_1s_1+\dots+l_ks_k=n-1}} \frac{(n-1)!}{l_1^{s_1}\dots l_k^{s_k}s_1!\dots s_k!}\times\\
&\int_{\mathbb{R}_+^{s_1+\dots+s_k+1}} |g_t^{(n-1)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k}}_{l_k})|^2\times\\
&\eta^{l_1-1}(t_1)\dots\eta^{l_k-1}(t_{s_1+\dots+s_k})dt_1\dots dt_{s_1+\dots+s_k}dt=\\
&n\sum_{\substack{k,l_j,s_j\in\mathbb{N}:\ j=1,\dots,k,\ l_1>\dots>l_k>1,\\ l_1s_1+\dots+l_ks_k+1=n}} \frac{n!}{l_1^{s_1}\dots l_k^{s_k}s_1!\dots s_k!}\times\\
&\int_{\mathbb{R}_+^{s_1+\dots+s_k+1}} |f^{(n)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k}}_{l_k},t)|^2\times\\
&\eta^{l_1-1}(t_1)\dots\eta^{l_k-1}(t_{s_1+\dots+s_k})dt_1\dots dt_{s_1+\dots+s_k}dt+\\
&n\sum_{\substack{k,l_j,s_j\in\mathbb{N}:\ j=1,\dots,k,\ l_1>\dots>l_k=1,\\ l_1s_1+\dots+l_{k-1}s_{k-1}+s_k+1=n}} \frac{n!}{l_1^{s_1}\dots l_{k-1}^{s_{k-1}}s_1!\dots(s_k+1)!(s_k+1)}\times\\
&\int_{\mathbb{R}_+^{s_1+\dots+s_k+1}} |f^{(n)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,t_{s_1+\dots+s_{k-1}+1},\dots,t_{s_1+\dots+s_k},t)|^2\times\\
&\eta^{l_1-1}(t_1)\dots\eta^{l_{k-1}-1}(t_{s_1+\dots+s_{k-1}})dt_1\dots dt_{s_1+\dots+s_k}dt\le n|F^{(n)}|^2_{ext}<\infty.
\end{aligned}
$$

Therefore the function $g_\cdot^{(n-1)}$ generates an element (an equivalence class) $G_\cdot^{(n-1)}\in\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}$. It is easy to see that $\widehat{G}^{(n-1)}=F^{(n)}$ (see Subsection 1.3): for the representative $g_\cdot^{(n-1)}\in G_\cdot^{(n-1)}$ which is defined by (14) $\widehat{g}^{(n-1)}(\cdot_1,\dots,\cdot_n)=g_{\cdot_n}^{(n-1)}(\cdot_1,\dots,\cdot_{n-1})\cdot h_n(\cdot_1,\dots,\cdot_n)=f^{(n)}(\cdot_1,\dots,\cdot_n)\in F^{(n)}$ because $f^{(n)}$ is a symmetric function described in the condition of statement (2). Set $G(\cdot):=\langle P_{n-1},G_\cdot^{(n-1)}\rangle$. Now $F=\int G(t)\widehat{d}M_t$, so, the condition of statement (1) is fulfilled.

2) ("(1)⇒(2)") Let the condition of statement (1) be fulfilled, i.e.,

$$\langle P_n,F^{(n)}\rangle=\int\langle P_{n-1},G_t^{(n-1)}\rangle\widehat{d}M_t,\quad G_\cdot^{(n-1)}\in\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}.$$

By definition of the extended stochastic integral it means that $F^{(n)}=\widehat{G}^{(n-1)}$, but an element $\widehat{G}^{(n-1)}\in\mathcal{H}_{ext}^{(n)}$ satisfies the condition from statement (2) by construction.

The carryover of the result to the general case is trivial; we note only that if $F\in(L^2)$ and satisfies the condition of statement (2) then the formally constructed integrand G belongs to $(L^2)\otimes\mathcal{H}$ because for each $n\in\mathbb{N}$ we have $|G_\cdot^{(n-1)}|^2_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}}\le n|F^{(n)}|^2_{ext}$ (for $n=1$, $G_\cdot^{(0)}=F^{(1)}\in\mathcal{H}=\mathcal{H}_{ext}^{(1)}$), therefore $\|G\|^2_{(L^2)\otimes\mathcal{H}}=\sum_{n=1}^\infty(n-1)!|G_\cdot^{(n-1)}|^2_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}}\le\sum_{n=1}^\infty n!|F^{(n)}|^2_{ext}\le\|F\|^2_{(L^2)}<\infty$. □

Remark 2.3. *Let $F \in (L^2)$ and be presentable in the form $F = \mathbf{E}F + \int \mathcal{G}(t)\widehat{d}M_t$, where $\mathcal{G}(\cdot) = \sum_{n=1}^{\infty}\langle P_{n-1}, \mathcal{G}_{\cdot}^{(n-1)}\rangle$, $\mathcal{G}_{\cdot}^{(n-1)} \in \mathcal{H}_{ext}^{(n-1)} \otimes \mathcal{H}$, is a formal series (i.e., this series can diverge in $(L^2)\otimes\mathcal{H}$) and $\int \mathcal{G}(t)\widehat{d}M_t$ is a formal stochastic integral, i.e., $\int \mathcal{G}(t)\widehat{d}M_t = \sum_{n=1}^{\infty}\langle P_n, \widehat{\mathcal{G}}^{(n-1)}\rangle$. As is easy to see, now for each $n \in \mathbb{N}$ $F^{(n)} = \widehat{\mathcal{G}}^{(n-1)}$ in $\mathcal{H}_{ext}^{(n)}$, therefore F satisfies the condition of statement (2) of Proposition 2.1 whence it follows that F can be presented in form (12) with an integrand $G \in (L^2)\otimes\mathcal{H}$ (note that $G \neq \mathcal{G}$, generally speaking). So, in what follows, in corresponding places we will write "F can be presented in form (12)" without the reminder that $G \in (L^2)\otimes\mathcal{H}$.*

Remark 2.4. *If $F = \langle P_n, F^{(n)}\rangle$, $n \in \mathbb{N}\backslash\{1\}$, cannot be presented in form (12), one still can define the function $g_{\cdot}^{(n-1)}$ by (14) and construct the corresponding element $\widehat{G}^{(n-1)} \in \mathcal{H}_{ext}^{(n)}$. But now $F^{(n)} \neq \widehat{G}^{(n-1)}$ in $\mathcal{H}_{ext}^{(n)}$ and $|\widehat{G}^{(n-1)}|_{ext} < |F^{(n)}|_{ext}$ (the norm $|F^{(n)} - \widehat{G}^{(n-1)}|_{ext}$ contains integrals by families of arguments for which h_n is equal to zero).*

Corollary. *If $F \in (L^2)$ and is presentable in form (12) then the kernels $G_{\cdot}^{(n-1)} \in \mathcal{H}_{ext}^{(n-1)} \otimes \mathcal{H}$ from decomposition (6) of an integrand can be constructed by representatives (14).*

2.3 Clark-Ocone type formulas

It is described above how for a representable in form (12) random variable $F \in (L^2)$ to reconstruct a corresponding integrand $G \in (L^2)\otimes\mathcal{H}$. But such a description is not convenient for applications. In this subsection we prove statements, in which an integrand G for a given F is presented in a more convenient for applications form.

We begin from some preparation. For $n \in \mathbb{N}\backslash\{1\}$ and $t_1, \dots, t_n \in \mathbb{R}_+$ set $\hbar_n(t_1, \dots, t_n) := nh_n(t_1, \dots, t_n)$, where the functions h_n are defined in (13); set also $\hbar_1 \equiv 1$. Further, for $G_{\cdot}^{(n)} \in \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$, $n \in \mathbb{Z}_+$, set

$$\widetilde{G}_{\cdot}^{(n)}(\cdot_1, \dots, \cdot_n) := \begin{cases} \frac{G_{\cdot}^{(n)}(\cdot_1, \dots, \cdot_n)}{\hbar_{n+1}(\cdot_1, \dots, \cdot_n, \cdot)}, & \text{if } \hbar_{n+1}(\cdot_1, \dots, \cdot_n, \cdot) \neq 0 \\ 0, & \text{if } \hbar_{n+1}(\cdot_1, \dots, \cdot_n, \cdot) = 0. \end{cases}$$

It is easy to see that $\widetilde{G}_{\cdot}^{(n)} \in \mathcal{H}_{ext}^{(n)} \otimes \mathcal{H}$ and

$$|\widetilde{G}_{\cdot}^{(n)}|_{\mathcal{H}_{ext}^{(n)}\otimes\mathcal{H}} \leq |G_{\cdot}^{(n)}|_{\mathcal{H}_{ext}^{(n)}\otimes\mathcal{H}}. \tag{15}$$

For $G \in (L^2)\otimes\mathcal{H}$ we define

$$(AG)(\cdot) := \sum_{n=0}^{\infty}\langle P_n, \widetilde{G}_{\cdot}^{(n)}\rangle,$$

where the kernels $\widetilde{G}_{\cdot}^{(n)}$ are constructed by the kernels $G_{\cdot}^{(n)}$ from decomposition (6) for G. It follows from estimate (15) that A is a linear *continuous* operator in $(L^2)\otimes\mathcal{H}$.

Theorem 1. *Let $F \in (L^2)$, be presentable in form (12) (see Proposition 2.1) and belongs to the domain of the Hida stochastic derivative (see (9)). Then the representation*

$$F = \mathbf{E}F + \int A\partial_t F\widehat{d}M_t \tag{16}$$

is valid, where $\int A\partial_t F\widehat{d}M_t := \int (A\partial_{\cdot} F)(t)\widehat{d}M_t$.

Remark 2.5. *In the classical Gaussian (and Poissonian) analysis one can reconstruct $F-\mathbf{E}F$ for differentiable by Hida $F \in (L^2)$ by using of the Clark-Ocone formula if $\partial_\cdot F$ is known. But in the Meixner white noise analysis it is not the case: now it is impossible to reconstruct even $\langle P_n, F^{(n)}\rangle$ $(n \in \mathbb{N}\backslash\{1\})$ if $\partial_\cdot\langle P_n, F^{(n)}\rangle$ is known, generally speaking, because different $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$ can coincide as elements of $\mathcal{H}_{ext}^{(n-1)} \otimes \mathcal{H}$. Nevertheless, for F satisfying the conditions of Theorem 1 $F - \mathbf{E}F$ can be reconstructed if $\partial_\cdot F$ is known. But for such a reconstruction one has to use the extended stochastic integral in Clark-Ocone type formula* (16) *because $A\partial_\cdot F$ can be nonintegrable by Itô.*

Proof. It is sufficient to prove the theorem for $F = \langle P_n, F^{(n)}\rangle$, $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}\backslash\{1\}$ (the cases $n = 0$ and $n = 1$ are trivial). Let us accept by definition $\frac{0}{0} := 0$. Using the definitions of the Hida stochastic derivative and of the operator A, we can write

$$A\partial_\cdot F = n\langle P_{n-1}, \widetilde{F}^{(n)}(\cdot)\rangle = n\Big\langle P_{n-1}, \frac{f^{(n)}(\cdot_1, \dots, \cdot_{n-1}, \cdot)}{\hbar_n(\cdot_1, \dots, \cdot_{n-1}, \cdot)}\Big\rangle = \Big\langle P_{n-1}, \frac{f^{(n)}(\cdot_1, \dots, \cdot_{n-1}, \cdot)}{h_n(\cdot_1, \dots, \cdot_{n-1}, \cdot)}\Big\rangle,$$

where $f^{(n)} \in F^{(n)} \in \mathcal{H}_{ext}^{(n)}$ is a *symmetric* function described in statement (2) of Proposition 2.1 (note that if for a family of arguments $t_1, \dots, t_{n-1}, t \in \mathbb{R}_+$ $h_n(t_1, \dots, t_{n-1}, t) = 0$ then $f^{(n)}(t_1, \dots, t_{n-1}, t) = 0$). But by construction of the kernels of the extended stochastic integral (see Subsection 1.3) we have now $\widehat{\frac{f^{(n)}}{h_n}} = f^{(n)} \in F^{(n)}$, whence $\int (A\partial_\cdot\langle P_n, F^{(n)}\rangle)(t)\widehat{d}M_t = \langle P_n, F^{(n)}\rangle$, which is what had to be proved. □

Corollary. *If $F \in (L^2)$, can be presented in form* (12) *and belongs to the domain of the Hida stochastic derivative then an integrand G from* (12) *can be presented in the form*

$$G(\cdot) = A\partial_\cdot F.$$

Formula (16) can be interpreted as a Clark-Ocone type formula in the Meixner white noise analysis, but this formula is not a direct analog of classical Clark-Ocone formula (1). In fact, if μ is the Gaussian or Poissonian measure then, as is easily seen, for $G \in (L^2) \otimes \mathcal{H}$ we have $(AG)(\cdot) = \sum_{n=0}^{\infty}\langle P_n, \frac{G_\cdot^{(n)}}{n+1}\rangle$, where $G_\cdot^{(n)} \in \mathcal{H}^{\widehat{\otimes} n} \otimes \mathcal{H}$ are the kernels from decomposition (6) for G. On the other hand, one can understand G as the family of functions $g_\alpha : \mathbb{R}_+ \to (L^2)$ ($\|g_\alpha\|_{(L^2)\otimes\mathcal{H}} < \infty$, $\alpha \in \Theta$—some set of indexes) that is defined by an arbitrary representative $g_\alpha \in G$ and is such that for arbitrary $\alpha_1, \alpha_2 \in \Theta$ $\|g_{\alpha_1} - g_{\alpha_2}\|_{(L^2)\otimes\mathcal{H}} = 0$. In this case $\mathbf{E}\{G(\cdot)|_{\mathcal{F}_\cdot}\} \in (L^2) \otimes \mathcal{H}$ is an equivalense class in $(L^2) \otimes \mathcal{H}$ that contains the family of functions $\mathbb{R}_+ \ni t \mapsto \mathbf{E}\{g_\alpha(t)|_{\mathcal{F}_t}\}$, $\alpha \in \Theta$, and even for G of form $G(\cdot) = \partial_\cdot F$, $F \in (L^2)$, we have $\mathbf{E}\{G(\cdot)|_{\mathcal{F}_\cdot}\} = \sum_{n=0}^{\infty}\langle P_n, G_\cdot^{(n)} 1_{[0,\cdot)^n}\rangle \neq (AG)(\cdot)$, generally speaking.

Let us obtain a direct analog of formula (1) in the Meixner white noise analysis. For $n \in \mathbb{N}$ and $t_1, \dots, t_n, t \in \mathbb{R}_+$ set

$$\chi_{n,t}(t_1, \dots, t_n) := \begin{cases} 1, \text{ if } \forall i \in \{1, \dots, n\} : \big(\forall j \in \{1, \dots, n\}\backslash\{i\}\ t_i \neq t_j\big)\ t_i < t, \\ 0, \text{ in other cases,} \end{cases}$$

i.e., $\chi_{n,t}(t_1, \dots, t_n) = 1$ if all t_i of the multiplicity one are smaller than t. For example, $\chi_{3,5}(6, 6, 4) = 1$ ($4 < 5$, 6 has the multiplicity two), but $\chi_{3,5}(6, 4, 4) = 0$ ($6 > 5$, 6 has the multiplicity one). Set also $\chi_{0,\cdot}\equiv 1$. For $F\in(L^2)$ and $t\in\mathbb{R}_+$ define an operator $\widetilde{\mathbf{E}}\{F|_{\mathcal{F}_t}\}\in(L^2)$ by setting

$$\widetilde{\mathbf{E}}\{F|_{\mathcal{F}_t}\}:=\sum_{n=0}^{\infty}\langle P_n,F^{(n)}\chi_{n,t}\rangle, \tag{17}$$

where $F^{(n)}\in\mathcal{H}_{ext}^{(n)}$ are the kernels from decomposition (4) for F. As is easily seen, we have $|F^{(n)}\chi_{n,t}|_{ext}\le|F^{(n)}|_{ext}$, therefore $\widetilde{\mathbf{E}}\{\circ|_{\mathcal{F}_t}\}$ is a linear continuous operator in (L^2).

Remark 2.6. *We use for the operator $\widetilde{\mathbf{E}}\{\circ|_{\mathcal{F}_t}\}$ the notation that is similar to the designation of a conditional expectation because these operators are similar in a sense: cf.* (17) *and* (11)*. Moreover, it is easy to see that in the Gaussian and Poissonian cases $\widetilde{\mathbf{E}}\{\circ|_{\mathcal{F}_t}\}=\mathbf{E}\{\circ|_{\mathcal{F}_t}\}$ because for $n\in\mathbb{N}$ $\chi_{n,t}=1_{[0,t)^n}$ in $\mathcal{H}^{\widehat{\otimes}n}$ (i.e., these two functions belong to the same equivalence class in this space).*

Theorem 2. *Let $F\in(L^2)$, be presentable in form* (12) *(see Proposition 2.1) and belong to the domain of the Hida stochastic derivative (see* (9)*). Then the representation*

$$F=\mathbf{E}F+\int\widetilde{\mathbf{E}}\{\partial_tF|_{\mathcal{F}_t}\}\widehat{d}M_t \tag{18}$$

is valid.

Remark 2.7. *If the kernels $F^{(n)}$ from decomposition* (4) *for F can be considered as elements of $\mathcal{H}^{\widehat{\otimes}n}$ (see Subsection 1.2) then formula* (18) *reduces to* (10).

Proof. It is sufficient to prove the theorem for $F=\langle P_n,F^{(n)}\rangle$, $F^{(n)}\in\mathcal{H}_{ext}^{(n)}$, $n\in\mathbb{N}\backslash\{1\}$ (the cases $n=0$ and $n=1$ are trivial). Let $f^{(n)}\in F^{(n)}$ and be a symmetric function described in statment (2) of Proposition 2.1. Then for almost all (with respect to the Lebesgue measure) $t\in\mathbb{R}_+$ $\partial_t\langle P_n,f^{(n)}\rangle=n\langle P_{n-1},f^{(n)}(t)\rangle$, $\widetilde{\mathbf{E}}\{\partial_t\langle P_n,f^{(n)}\rangle|_{\mathcal{F}_t}\}=n\langle P_{n-1},f^{(n)}(t)\chi_{n-1,t}\rangle$, and

$$\int\widetilde{\mathbf{E}}\{\partial_t\langle P_n,F^{(n)}\rangle|_{\mathcal{F}_t}\}\widehat{d}M_t=\int\widetilde{\mathbf{E}}\{\partial_t\langle P_n,f^{(n)}\rangle|_{\mathcal{F}_t}\}\widehat{d}M_t=n\langle P_n,\widehat{f^{(n)}(\cdot)\chi_{n-1,\cdot}}\rangle.$$

Therefore we have to show that $n\widehat{f^{(n)}(\cdot)\chi_{n-1,\cdot}}\in F^{(n)}$ in $\mathcal{H}_{ext}^{(n)}$. Using the construction of $\widehat{f^{(n)}(\cdot)\chi_{n-1,\cdot}}$, (3), nonatomicity of the Lebesgue measure and the fact that $f^{(n)}$ is a symmetric function satisfying the condition from statement (2) of Proposition 2.1, we obtain

$$\begin{gathered}
\left|F^{(n)}-n\widehat{f^{(n)}(\cdot)\chi_{n-1,\cdot}}\right|_{ext}^2=\left|f^{(n)}-n\widehat{f^{(n)}(\cdot)\chi_{n-1,\cdot}}\right|_{ext}^2=\\
\sum_{\substack{k,l_j,s_j\in\mathbb{N}:\ j=1,\dots,k,\ l_1>l_2>\dots>l_k=1,\\ l_1s_1+\dots+l_{k-1}s_{k-1}+s_k=n}}\frac{n!}{l_1^{s_1}\dots l_{k-1}^{s_{k-1}}s_1!\dots s_k!}\times\\
\int_{\mathbb{R}_+^{s_1+\dots+s_k}}\Big|f^{(n)}(\underbrace{t_1,\dots,t_1}_{l_1},\dots,\underbrace{t_{s_1+\dots+s_{k-1}},\dots,t_{s_1+\dots+s_{k-1}}}_{l_{k-1}},t_{s_1+\dots+s_{k-1}+1},\dots,t_{s_1+\dots+s_k})\times\\
\big[1-1_{\{t_{s_1+\dots+s_{k-1}+1}<t_{s_1+\dots+s_k},t_{s_1+\dots+s_{k-1}+2}<t_{s_1+\dots+s_k},\dots,t_{s_1+\dots+s_k-1}<t_{s_1+\dots+s_k}\}}-\\
1_{\{t_{s_1+\dots+s_k}<t_{s_1+\dots+s_k-1},t_{s_1+\dots+s_{k-1}+1}<t_{s_1+\dots+s_k-1},\dots,t_{s_1+\dots+s_k-2}<t_{s_1+\dots+s_k-1}\}}-\dots-\\
1_{\{t_{s_1+\dots+s_{k-1}+2}<t_{s_1+\dots+s_{k-1}+1},t_{s_1+\dots+s_{k-1}+3}<t_{s_1+\dots+s_{k-1}+1},\dots,t_{s_1+\dots+s_k}<t_{s_1+\dots+s_{k-1}+1}\}}\big]\Big|^2\times\\
\eta^{l_1-1}(t_1)\dots\eta^{l_{k-1}-1}(t_{s_1+\dots+s_{k-1}})dt_1\dots dt_{s_1+\dots+s_k}=0
\end{gathered}$$

(for *different* $t_{s_1+\cdots+s_{k-1}+1}, \ldots, t_{s_1+\cdots+s_k}$ one and only one indicator in this calculation is equal to one; other cases can be ignored because

$$m^{\otimes s_k}\big(\{t_{s_1+\cdots+s_{k-1}+1}, \ldots, t_{s_1+\cdots+s_k}\} : \\ \exists i, j \in \{s_1 + \cdots + s_{k-1} + 1, \ldots, s_1 + \cdots + s_k\} : i \neq j, t_i = t_j\big) = 0,$$

where m is the Lebesgue measure on $\mathbb{R}_+$). $\square$

Remark 2.8. *One can introduce a linear continuous operator* $\widetilde{\mathbf{E}}\big\{\circ(\cdot)|_{\mathcal{F}_\cdot}\big\}$ *in* $(L^2)\otimes\mathcal{H}$ *by setting (cf.* (17)*)*

$$\widetilde{\mathbf{E}}\big\{G(\cdot)|_{\mathcal{F}_\cdot}\big\} := \sum_{n=0}^{\infty}\langle P_n, G_\cdot^{(n)}\chi_{n,\cdot}\rangle, \tag{19}$$

where $G_\cdot^{(n)} \in \mathcal{H}_{ext}^{(n)}\otimes\mathcal{H}$ *are the kernels from decomposition* (6) *for* G*. In this case formula* (18) *holds true if we accept by definition* $\int \widetilde{\mathbf{E}}\big\{\partial_t F|_{\mathcal{F}_t}\big\}\widehat{d}M_t := \int \widetilde{\mathbf{E}}\big\{\partial_\cdot F|_{\mathcal{F}_\cdot}\big\}(t)\widehat{d}M_t$ *(cf. Theorem 1). Note that if* $G \in (L^2)\otimes\mathcal{H}$ *and* $g \in G$ *is a representative of* G *then the function* $\mathbb{R}_+ \ni t \mapsto \widetilde{\mathbf{E}}\big\{g(t)|_{\mathcal{F}_t}\big\} \in (L^2)$ *(see* (17)*) generates the equivalence class in* $(L^2)\otimes\mathcal{H}$ *that coincides with* $\widetilde{\mathbf{E}}\big\{G(\cdot)|_{\mathcal{F}_\cdot}\big\}$ *(see* (19)*).*

Remark 2.9. *Clark-Ocone type formulas* (16) *and* (18) *were proved under a very restrictive assumption that a random variable* $F \in (L^2)$ *is differentiable by Hida. But one can easily avoid this restriction considering* $\partial_\cdot$ *as a linear continuous operator acting from* (L^2) *to* $(L^2)_{-1}^0 \otimes \mathcal{H}$*, where* $(L^2)_{-1}^0$ *is the so-called parametrized Kondratiev-type space of regular generalized functions [13], and introducing* A *and* $\widetilde{\mathbf{E}}\big\{\circ|_{\mathcal{F}_t}\big\}$ *as linear continuous operators in* $(L^2)_{-1}^0\otimes\mathcal{H}$ *and* $(L^2)_{-1}^0$ *correspondingly by analogy with definitions given above.*

As we can see, the use of the extended stochastic integral and of special operators in Clark-Ocone type formulas is stipulated by properties of the generalized Meixner measure. Nevertheless, in some particular cases one can use the Itô stochastic integral and the conditional expectation. Let us consider the question about this possibility in more details.

Theorem 3. *Let* $F \in (L^2)$ *and belong to the domain of the Hida stochastic derivative (see* (9)*). Then the following statements are equivalent:*

(1) F *can be presented in form* (10)*;*

(2) for each $n \in \mathbb{N}\backslash\{1\}$ *the kernel* $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$ *from decomposition* (4) *for* F *has a representative* $f^{(n)} \in F^{(n)}$ *such that* $f^{(n)}(t_1, \ldots, t_n) = 0$ *if there exist* $i, j \in \{1, \ldots, n\}$, $i \neq j$, *such that* $\max\{t_1, \ldots, t_n\} = t_i = t_j$ *(i.e., if the multiplicity of maximal* $t_\cdot \in \{t_1, \ldots, t_n\}$ *is greater than one).*

Remark 2.10. *It is easy to see, if for some* $F \in (L^2)$ *the condition of statement (2) of this theorem is fulfilled (for example, it is so in the case* $\eta = 0$ *(see* (2)*)) then the condition of statement (2) of Proposition 2.1 is fulfilled too.*

Proof. It is sufficient to prove the theorem for $F = \langle P_n, F^{(n)}\rangle$, $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}\backslash\{1\}$.

1) ("(2)⇒(1)") Let $f^{(n)}$ be a representative of $F^{(n)}$ that is described in the condition of statement (2). Without loss of generality one can assume that $f^{(n)}$ is a symmetric function. Using (11), properties of the extended stochastic integral (see Subsection 1.3), and the fact that if $Pr1_{[0,t_n)^{n-1}}(t_1,\dots,t_{n-1}) = 0$ then $f^{(n)}(t_1,\dots,t_n) = 0$ by the condition from statement (2) (because $Pr1_{[0,t_n)^{n-1}}(t_1,\dots,t_{n-1}) = 0$ if and only if the multiplicity of maximal $t_\cdot \in \{t_1,\dots,t_n\}$ is greater than one), we obtain

$$\int \mathbf{E}\{\partial_t F|_{\mathcal{F}_t}\}\widehat{d}M_t = \int \mathbf{E}\{\partial_t\langle P_n, f^{(n)}\rangle|_{\mathcal{F}_t}\}\widehat{d}M_t = \int n\langle P_{n-1}, f^{(n)}(t)1_{[0,t)^{n-1}}\rangle\widehat{d}M_t =$$
$$\langle P_n, nf^{(n)}Pr1_{[0,\cdot_n)^{n-1}}(\cdot_1,\dots,\cdot_{n-1})\rangle = \langle P_n, f^{(n)}\rangle = F.$$

2) ("(1)⇒(2)") If $F = \langle P_n, F^{(n)}\rangle$ can be presented in form (10) then, as is easy to calculate, $n\widehat{F^{(n)}(\cdot)1_{[0,\cdot)^{n-1}}} = F^{(n)}$. But by construction the equivalence class $n\widehat{F^{(n)}(\cdot)1_{[0,\cdot)^{n-1}}} \in \mathcal{H}_{ext}^{(n)}$ contains a function $f^{(n)}$ that satisfies the condition from statement (2): one can consider a *symmetric* function $\widetilde{f}^{(n)} \in F^{(n)}$ in $\mathcal{H}_{ext}^{(n)}$ and set

$$f^{(n)}(t_1,\dots,t_n) := \begin{cases} \widetilde{f}^{(n)}(t_1,\dots,t_n), & \text{if } Pr1_{[0,t_n)^{n-1}}(t_1,\dots,t_{n-1}) \neq 0 \\ 0, & \text{if } Pr1_{[0,t_n)^{n-1}}(t_1,\dots,t_{n-1}) = 0. \end{cases}$$

□

Proposition 2.2. *Let $F \in (L^2)$, belong to the domain of the Hida stochastic derivative (see* (9)*) and be presentable in form* (10) *(see Theorem 3). Then $\widetilde{\mathbf{E}}\{\partial_\cdot F|_{\mathcal{F}_\cdot}\} = \mathbf{E}\{\partial_\cdot F|_{\mathcal{F}_\cdot}\}$ in $(L^2)\otimes\mathcal{H}$ (see Remark 2.8).*

Proof. It is sufficient to prove the statement for $F = \langle P_n, F^{(n)}\rangle$, $F^{(n)} \in \mathcal{H}_{ext}^{(n)}$, $n \in \mathbb{N}\backslash\{1\}$ (the cases $n = 0$ and $n = 1$ are trivial). Let $f^{(n)}$ be a representative of $F^{(n)}$ that is described in the condition of statement (2) of Theorem 3. It is sufficient to show that $f^{(n)}(\cdot)\chi_{n-1,\cdot} = f^{(n)}(\cdot)1_{[0,\cdot)^{n-1}}$ in $\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}$. Let $t_1,\dots,t_{n-1},t \in \mathbb{R}_+$ and be such that $f^{(n)}(t_1,\dots,t_{n-1},t)$ is well-defined. As is easy to see, if $\chi_{n-1,t}(t_1,\dots,t_{n-1}) - 1_{[0,t)^{n-1}}(t_1,\dots,t_{n-1}) \neq 0$ tnen the multiplicity of $\max\{t_1,\dots,t_{n-1},t\}$ is greater then one, but in this case $f^{(n)}(t_1,\dots,t_{n-1},t) = 0$. So, in any case $f^{(n)}(t_1,\dots,t_{n-1},t)[\chi_{n-1,t}(t_1,\dots,t_{n-1}) - 1_{[0,t)^{n-1}}(t_1,\dots,t_{n-1})] = 0$ and therefore $|f^{(n)}(\cdot)[\chi_{n-1,\cdot} - 1_{[0,\cdot)^{n-1}}]|_{\mathcal{H}_{ext}^{(n-1)}\otimes\mathcal{H}} = 0$, which is what had to be proved. □

REFERENCES

1. Кабанов Ю.М. *Расширенные стохастические интегралы* // Теория вероятностей и ее приложения. — 1975. — Т.20, №4. — С. 725–737.

2. Кабанов Ю.М., Скороход А.В. *Расширенные стохастические интегралы* // Материалы школы–семинара по теории случайных процессов, Вильнюс, Ин-т физики и математики. — 1975. — Т.I. — С. 123–167.

3. Скороход А.В. *Об обобщении стохастического интеграла* // Теория вероятностей и ее приложения. — 1975. — Т.20, №2. — С. 223–238.

4. Aase K., Oksendal B., Privault N., Uboe J. *White noise generalizations of the Clark-Haussmann-Ocone theorem with application to mathematical finance*, Finance Stochastics, **4**, 4 (2000), 465–496.

5. Benth F.E., Di Nunno G., Lokka A., Oksendal B., Proske F. *Explicit representation of the minimal variance portfolio in markets driven by Lévy processes*, Math. Finance, **13**, 1 (2003), 55–72.

6. Clark J.M. *The representation of functionals of Brownian motion by stochastic integrals*, Ann. Math. Stat., **41**, 4 (1970), 1282–1295.

7. De Faria M., Oliveira M.J., Streit L. *A generalized Clark-Ocone formula*, Random Oper. Stoch. Equ., **8**, 2 (2000), 163–174.

8. Di Nunno G., Oksendal B., Proske F., Malliavin calculus for Lévy processes with applications to finance. Universitext. Springer-Verlag, Berlin, 2009. — XIV+413 p.

9. Di Nunno G., Oksendal B., Proske F. *White noise analysis for Lévy processes*, J. Funct. Anal., **206**, 1 (2004), 109–148.

10. Es-Sebaiy K., Tudor C.A. *Lévy processes and Itô-Skorokhod integrals*, Theory Stoch. Process., **14**, 2 (2008), 10–18.

11. Gelfand I.M., Vilenkin N.Ya., Generalized Functions: Vol. 4. Applications of harmonic analysis. Academic Press, New York–London, 1964. — XIV+384 p.

12. Grecksch W., Roth C., Anh V.V. *Q-fractional Brownian motion in infinite dimensions with application to fractional Black-Scholes market*, Stoch. Anal. Appl., **27**, 1 (2009), 149–175.

13. Kachanovsky N.A. *An extended stochastic integral and a Wick calculus on parametrized Kondratiev-type spaces of Meixner white noise*, Infin. Dimens. Anal. Quantum Probab. Relat. Top., **11**, 4 (2008), 541–564.

14. Kachanovsky N.A. *Generalized stochastic derivatives on a space of regular generalized functions of Meixner white noise*, Meth. Func. Anal. and Top., **14**, 1 (2008), 32–53.

15. Kachanovsky N.A. *Generalized stochastic derivatives on parametrized spaces of regular generalized functions of Meixner white noise*, Meth. Func. Anal. and Top., **14**, 4 (2008), 334–350.

16. Kachanovsky N.A. *On an extended stochastic integral and the Wick calculus on the connected with the generalized Meixner measure Kondratiev-type spaces*, Meth. Func. Anal. and Top., **13**, 4 (2007), 338–379.

17. Kachanovsky N.A., Tesko V.A. *Stochastic integral of Hitsuda-Skorohod type on the extended Fock space*, Ukr. Math. J., **61**, 6 (2009), 873–907.

18. Karatzas I., Ocone D. *A generalized Clark representation formula, with application to optimal portfolios*, Stochastics Rep., **34**, 3-4 (1991), 187–220.

19. Karatzas I., Ocone D., Li, J. *An extension of Clark's formula*, Stochastics Rep., **37**, 3 (1991), 127–131.

20. Lokka A. *Martingale representation of functionals of Lévy Processes*, Stoch. Anal. Appl., **22**, 4 (2004), 867–892.

21. Lokka A., Martingale Representation, Chaos Expansion and Clark-Ocone Formulas. Research Report, Centre for Mathematical Physics and Stochastics, University of Aarhus, Denmark, 22, 1999. — 24 p.

22. Maas J., van Neerven J. *A Clark-Ocone formula in UMD Banach spaces*, Electron. Commun. Probab., 13 (2008), 151–164.

23. Peccati G., Taqqu M.S. *Stable convergence of generalized L^2 stochastic integrals and the principle of conditioning*, Electron. J. Probab., **12**, 15 (2007), 447–480.

24. Nualart D., Schoutens W. *Chaotic and predictable representations for Lévy processes*, Stochastic Proc. Appl., **90**, 1 (2000), 109–122.

25. Ocone D. *Malliavin's calculus and stochastic integral: representation of functionals of diffusion processes*, Stochastics, 12, 3-4 (1984), 161–185.

26. Osswald H. *Malliavin calculus on extensions of abstract Wiener spaces*, J. Math. Kyoto Univ., **48**, 2 (2008), 239–263.

27. Rodionova I.V. *Analysis connected with generating functions of exponential type in one and infinite dimensions*, Meth. Func. Anal. and Top., **11**, 3 (2005), 275–297.

28. Schoutens W., Stochastic processes and orthogonal polynomials. Lecture Notes in Statistics 146. Springer, New York, 2000. — XIV+163 p.

29. Skorohod A.V., Integration in Hilbert Space. Springer, New York–Heidelberg, 1974. — XII+177 p.

30. Zhang Xi. *Clark-Ocone formula and variational representation for Poisson functionals*, An. Probab., **37**, 2 (2009), 506–529.

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У класичному гауссівському аналізі формула Кларка-Окона дозволяє відтворити підінтегральну функцію, якщо відомий стохастичний інтеграл Іто. Цю формулу можна записати у вигляді

$$F = \mathbf{E}F + \int \mathbf{E}\big\{\partial_t F|_{\mathcal{F}_t}\big\} dW_t,$$

де функція (випадкова величина) F є квадратично інтегровною за гауссівською мірою та диференційовною за Хідою; $\mathbf{E}$ — математичне сподівання; $\mathbf{E}\big\{\circ|_{\mathcal{F}_t}\big\}$ — умовне математичне сподівання відносно повної σ-алгебри $\mathcal{F}_t$, породженої вінерівським процесом W до моменту часу t; $\partial_\cdot F$ — похідна Хіди F; $\int \circ(t)dW_t$ — стохастичний інтеграл Іто за вінерівським процесом.

У цій статті ми пояснюємо як відтворити підінтегральну функцію у випадку, коли замість гауссівської міри розглядається так звана узагальнена міра Майкснера μ (в залежності від параметрів μ може бути гауссівською, пуассонівською, гамма мірою та ін.), та отримуємо відповідні формули типу Кларка-Окона.

Качановский Н.А. *Формулы типа Кларка-Окона в майкснеровском анализе белого шума* // Карпатские математические публикации. — 2011. — Т.3, №1. — С. 56–72.

В классическом гауссовском анализе формула Кларка-Окона позволяет восстановить подынтегральную функцию, если известен стохастический интеграл Ито. Эту формулу можно записать в виде

$$F = \mathbf{E}F + \int \mathbf{E}\big\{\partial_t F|_{\mathcal{F}_t}\big\} dW_t,$$

где функция (случайная величина) F квадратично интегрируема по гауссовской мере и дифференцируема по Хиде; $\mathbf{E}$ — математическое ожидание; $\mathbf{E}\big\{\circ|_{\mathcal{F}_t}\big\}$ — условное математическое ожидание относительно полной σ-алгебры $\mathcal{F}_t$, порожденной винеровским процессом W до момента времени t; $\partial_\cdot F$ — производная Хиды F; $\int \circ(t)dW_t$ — стохастический интеграл Ито по винеровскому процессу.

В этой статье мы объясняем как восстановить подынтегральную функцию в случае, когда вместо гауссовской меры рассматривается так называемая обобщенная мера Майкснера μ (в зависимости от параметров μ может быть гауссовской, пуассоновской, гамма мерой и др.), и получаем соответствующие формулы типа Кларка-Окона.